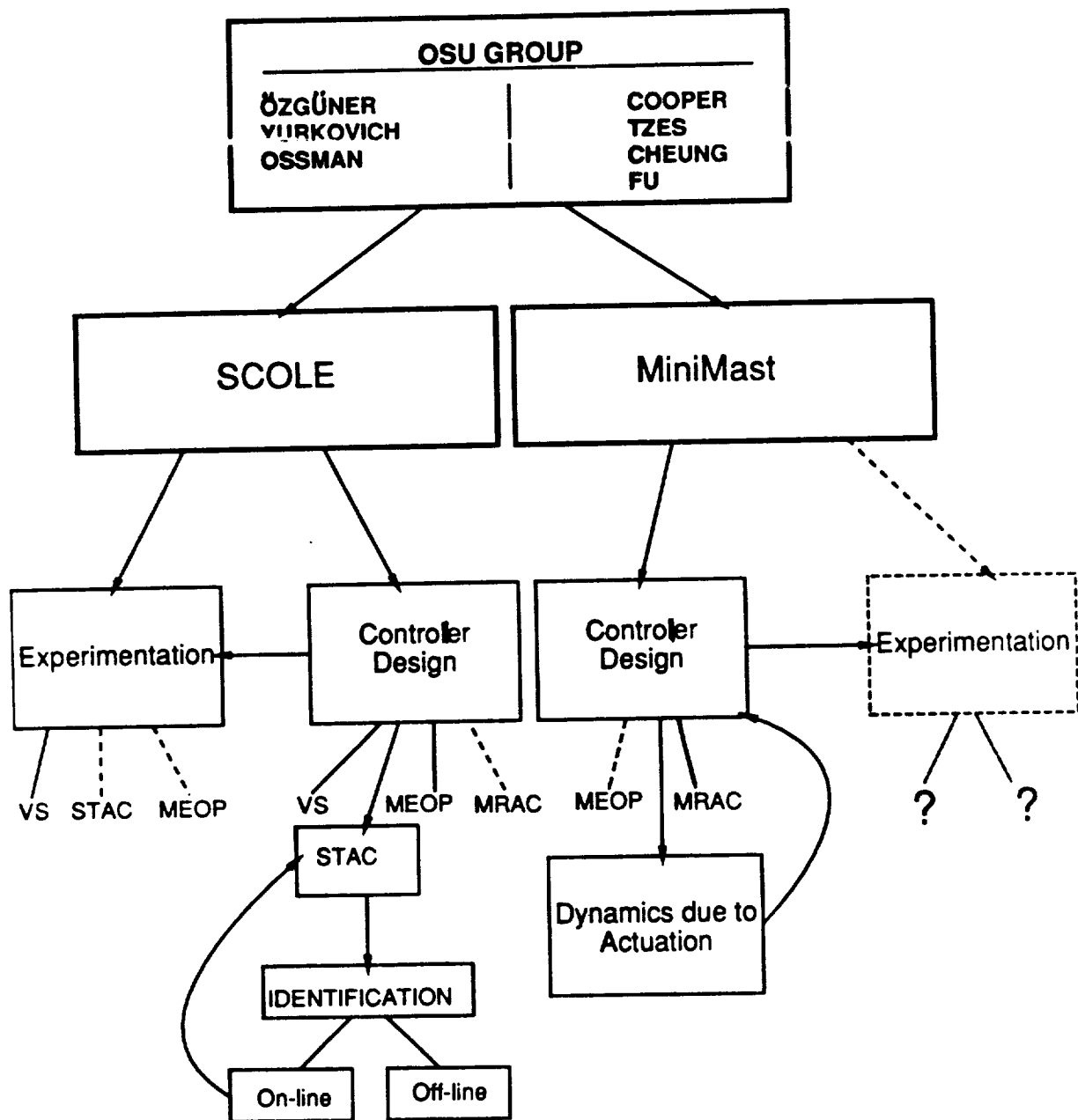


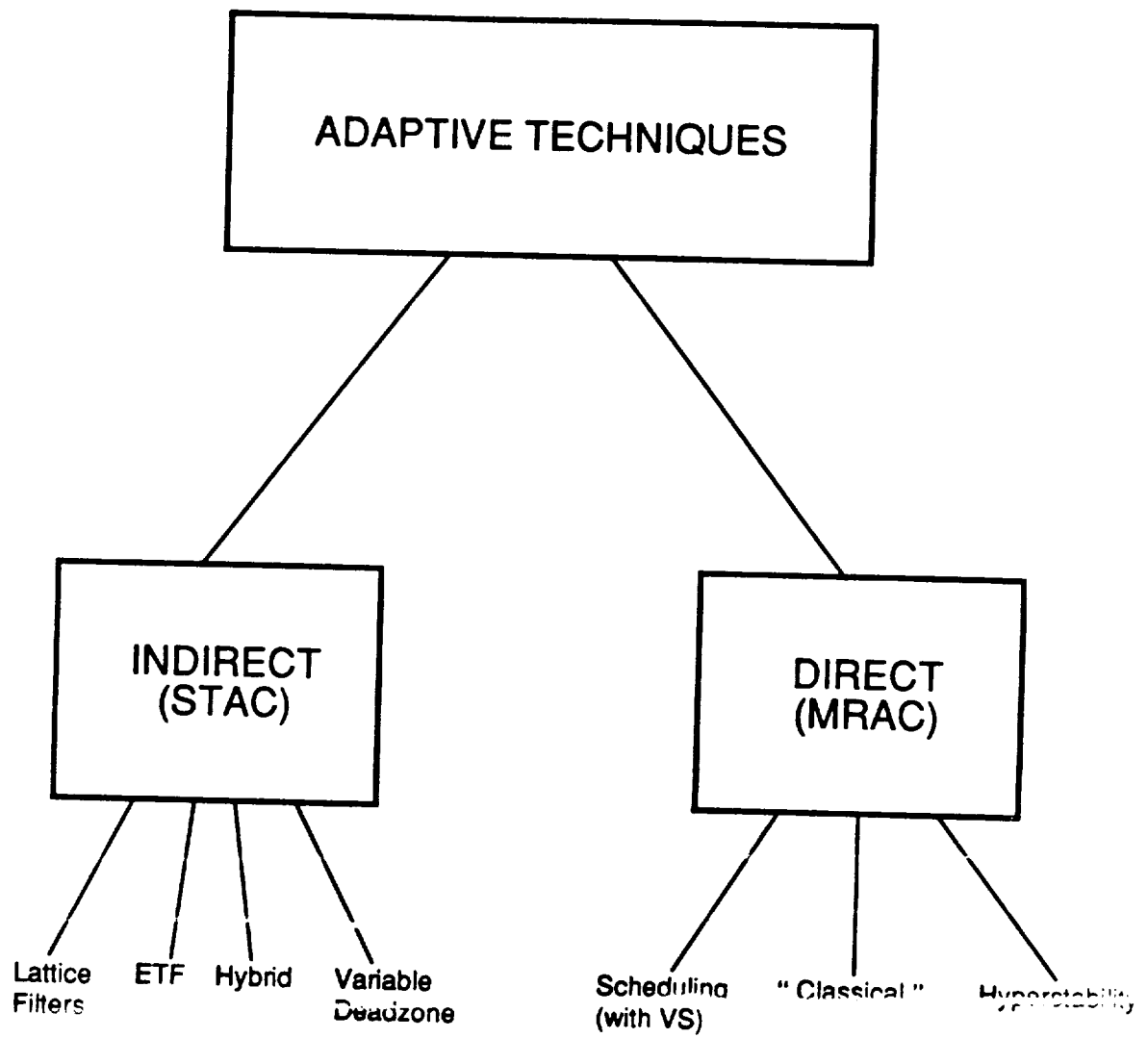
**Control Design Approaches  
for  
LaRC Experiments**

**Steve Yurkovich  
Ümit Özgüner**

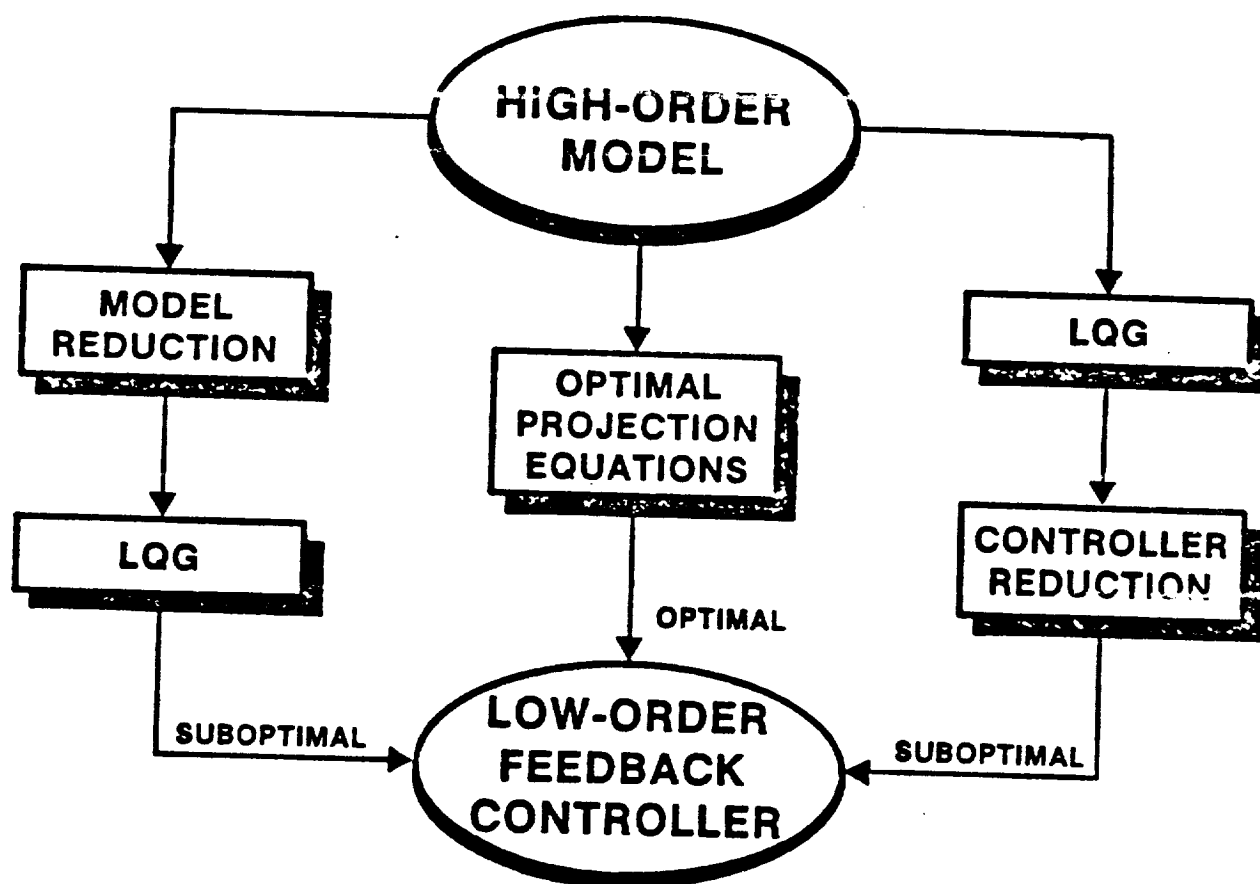
*The Ohio State University  
Columbus, OH*

*The 4th Annual SCOLE Workshop*





## Direct Fixed-Order Compensator Design



Given ...

$$\dot{x} = Ax + Bu + w_1$$

$$y = Cx + w_2 \quad ,$$

... design an  $n_c^{th}$  order *robust*, zero set-point compensator

$$\dot{x}_c = A_c x + Fy$$

$$u = -Kx_c$$

to minimize

$$J = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau (x^T(t)R_1x(t) + u^T(t)R_2u(t))dt \quad ,$$

## LQG Solution

$$K = R_2^{-1} B^T P \quad ,$$

$$F = Q C^T V_2^{-1} \quad ,$$

$$A_c = A - BK - FC \quad ,$$

$P$  and  $Q$  positive definite solutions to

$$PA + A^T P + R_1 - PBR_2^{-1}B^T P = 0 \quad ,$$

$$QA^T + AQ + V_1 - QC^T V_2^{-1} CQ = 0 \quad .$$

$$\begin{aligned}
0 &= PA_s + A_s^T P + \sum_{i=1}^{\mu} A_i^T P A_i - P_s^T R_{2s}^{-1} P_s + R_1 \\
&\quad + \sum_{i=1}^{\mu} (A_i - Q_s V_{2s}^{-1} C_i)^T \hat{P} (A_i - Q_s V_{2s}^{-1} C_i) + \tau_{\perp}^T P B R_2^{-1} B^T P \tau_{\perp} \\
0 &= A_s Q + Q A_s^T + \sum_{i=1}^{\mu} A_i Q A_i^T - Q_s V_{2s}^{-1} Q_s^T + V_1 \\
&\quad + \sum_{i=1}^{\mu} (A_i - B_i R_{2s}^{-1} P_s) \hat{Q} (A_i - B_i R_{2s}^{-1} P_s)^T + \tau_{\perp} Q C^T V_2^{-1} C Q \tau_{\perp}^T \\
0 &= \hat{P} A_{Qs} + A_{Qs}^T \hat{P} + P_s^T R_{2s}^{-1} P_s - \tau_{\perp}^T P B R_2^{-1} B^T P \tau_{\perp} \\
0 &= A_{Ps} \hat{Q} + \hat{Q} A_{Ps}^T + Q_s V_{2s}^{-1} Q_s^T - \tau_{\perp} Q C^T V_2^{-1} C Q \tau_{\perp}^T
\end{aligned}$$

## **Application to SCOLE**

- Reflector Panel
- Objective: Vibration Damping
- 3 inputs (reaction wheels at hub)
- 5 outputs (gyros at hub, accelerometers at reflector center)
- 10 modes



## MEOP procedure

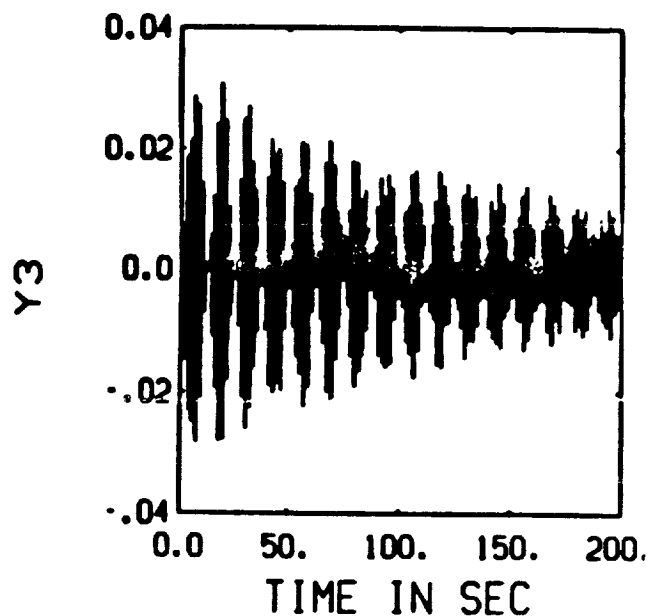
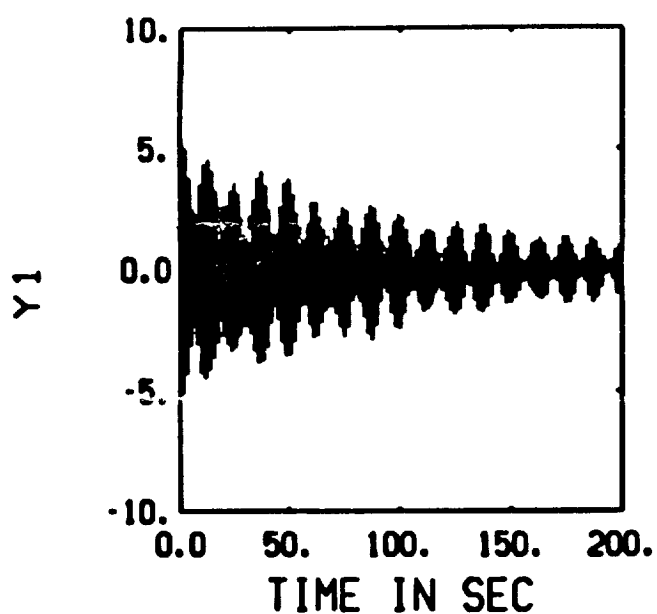
- Robustification with respect to modal frequencies
- Robustness measure:  $\epsilon$
- Results:

Order	$ \epsilon $	$\frac{d\omega}{\omega}$ (%)	Cost
20	0.0138	-20 to +4	0.229
12	0.0141	-30 to +20	0.231
10	0.0153	-45 to +30	0.231
8	0.0140	-9 to +30	0.235

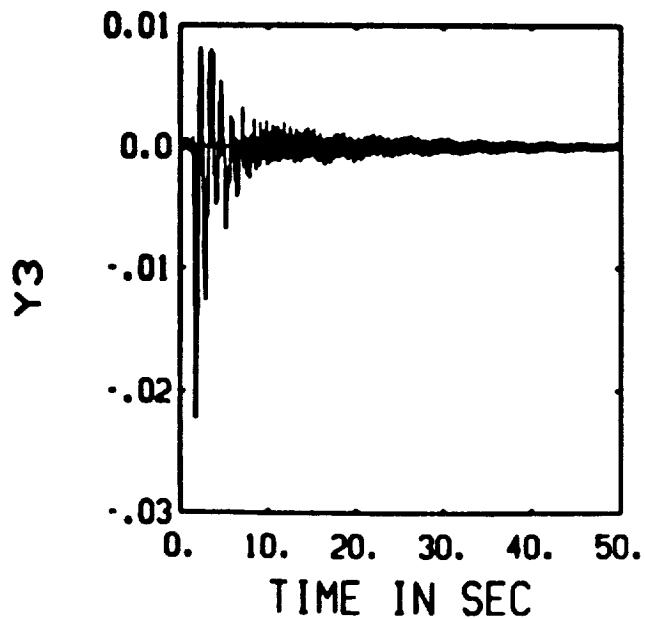
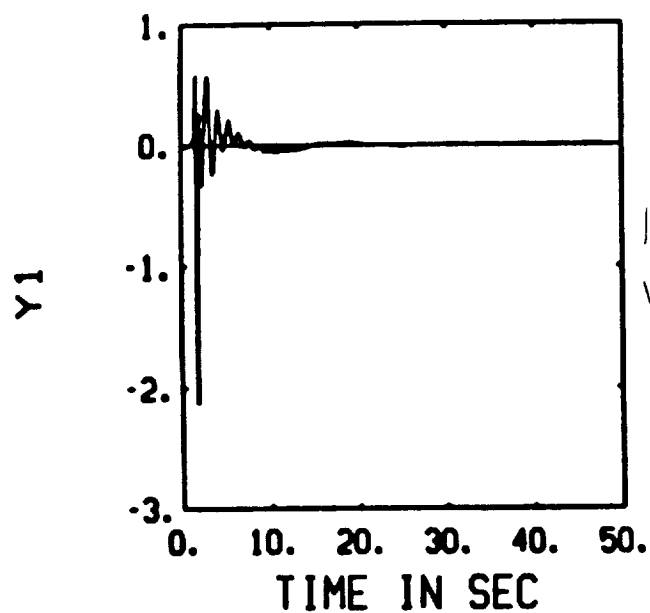
### Optimal Projection Design

Order	$ \epsilon $	$\frac{d\omega}{\omega}$ (%)	Cost
20	0.0148	-25 to +40	0.407
12	0.0156	-50 to +50	0.311
10	0.0154	-50 to +50	0.319
8	0.0154	-40 to +40	0.322

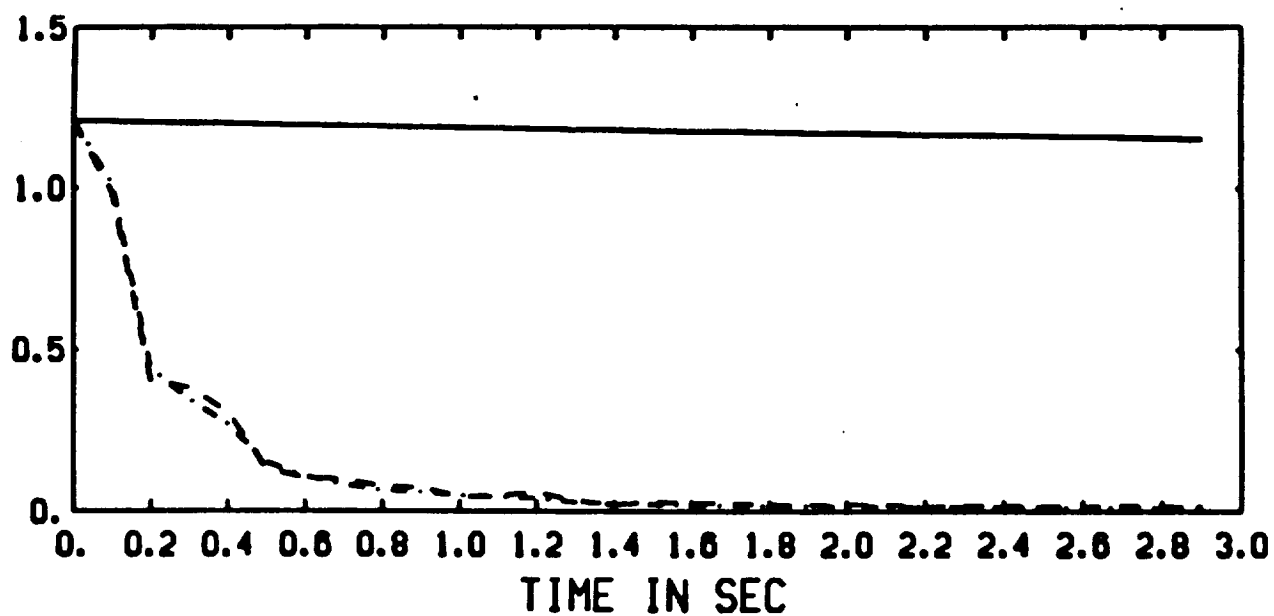
### MEOP Designs



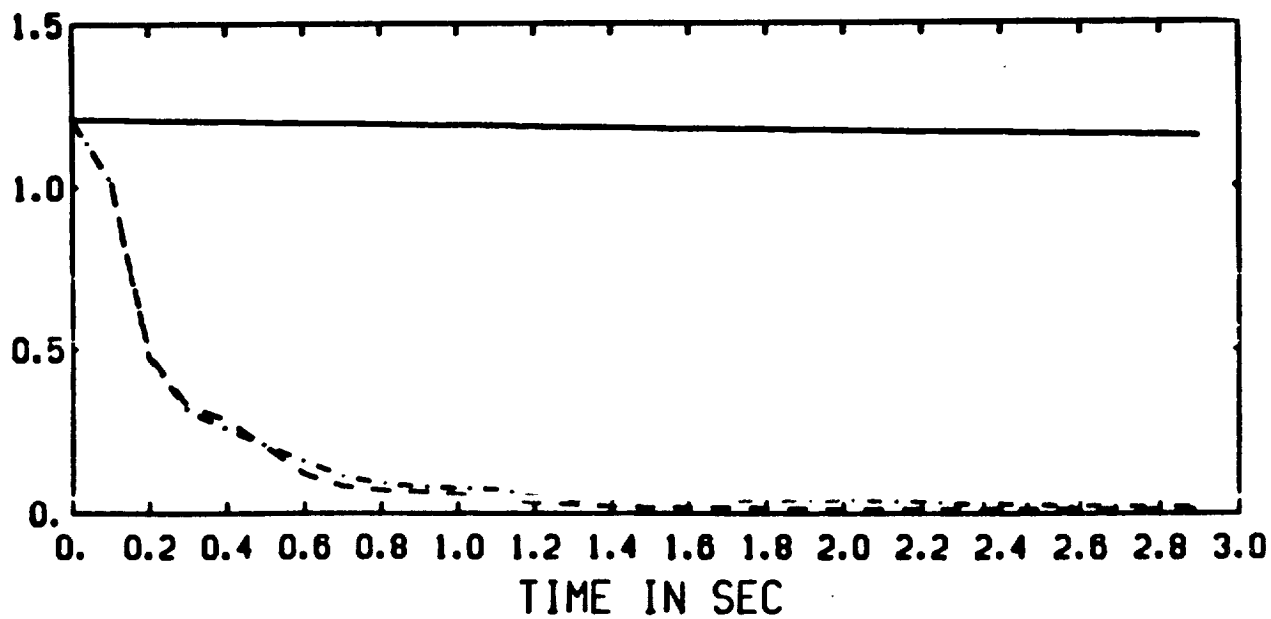
Open loop outputs.



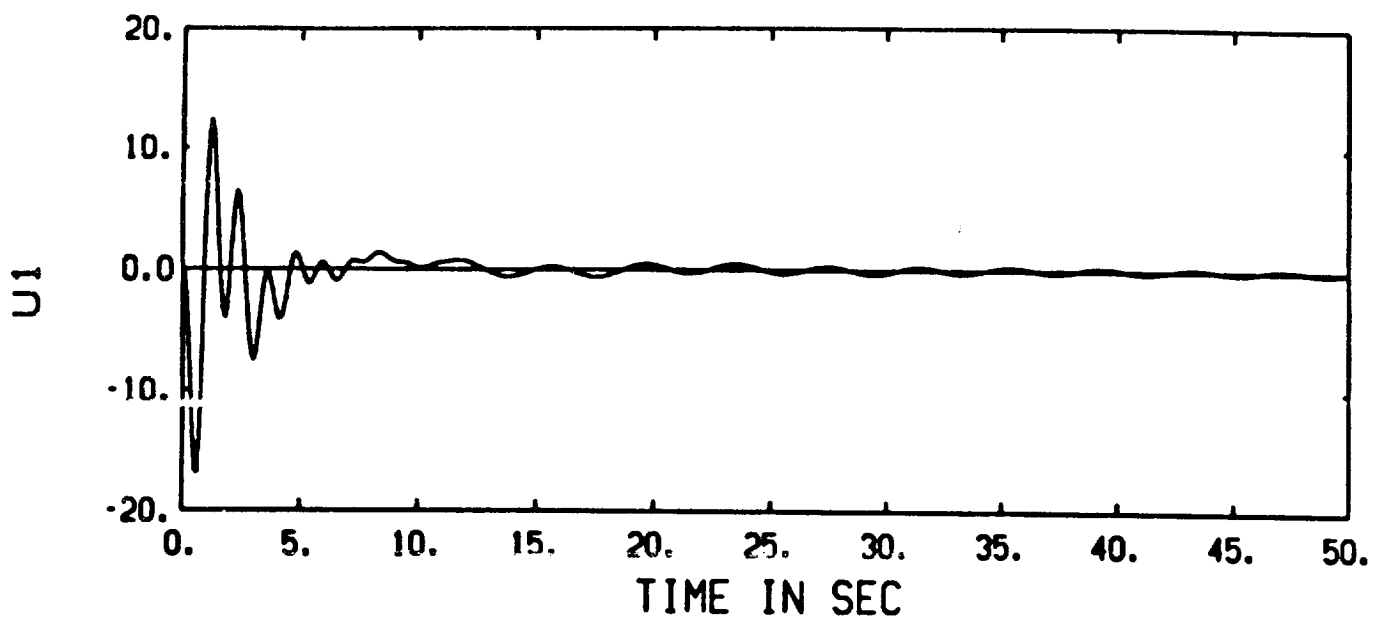
Closed loop outputs.



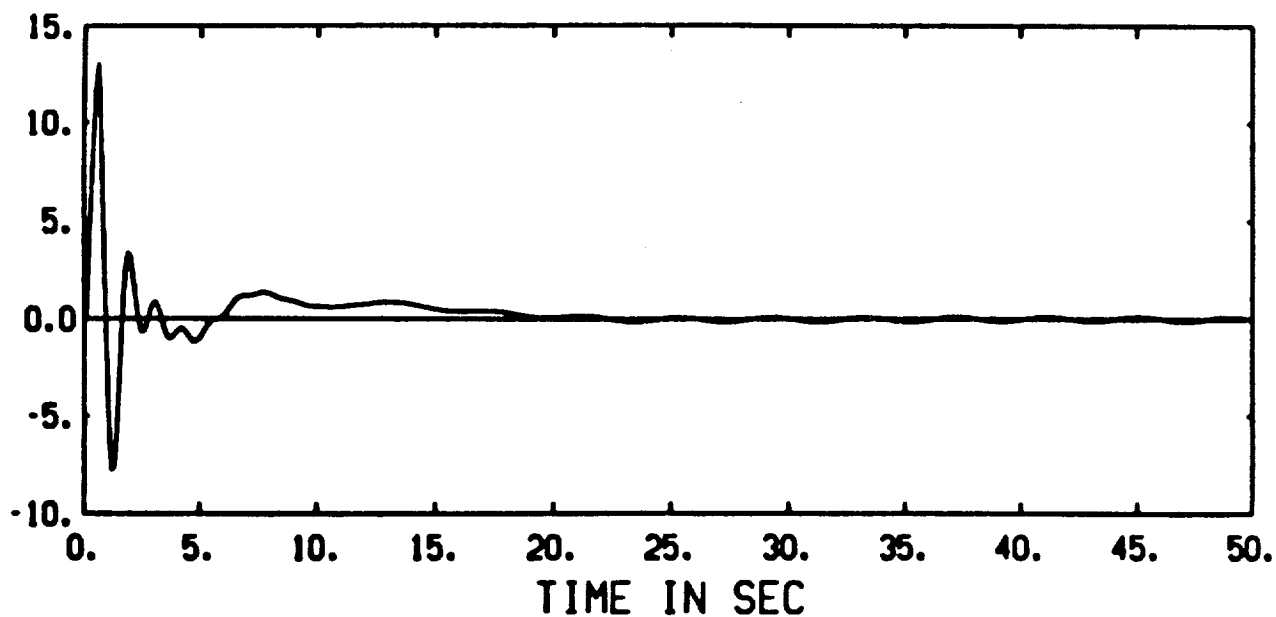
Vibrational energy profile  
(full order designs)



Vibrational energy profile  
(reduced-order designs)



Torque input on x-axis at the hub.



Torque input on y-axis at the hub.

## **Model Reference Adaptive Control (MRAC)**

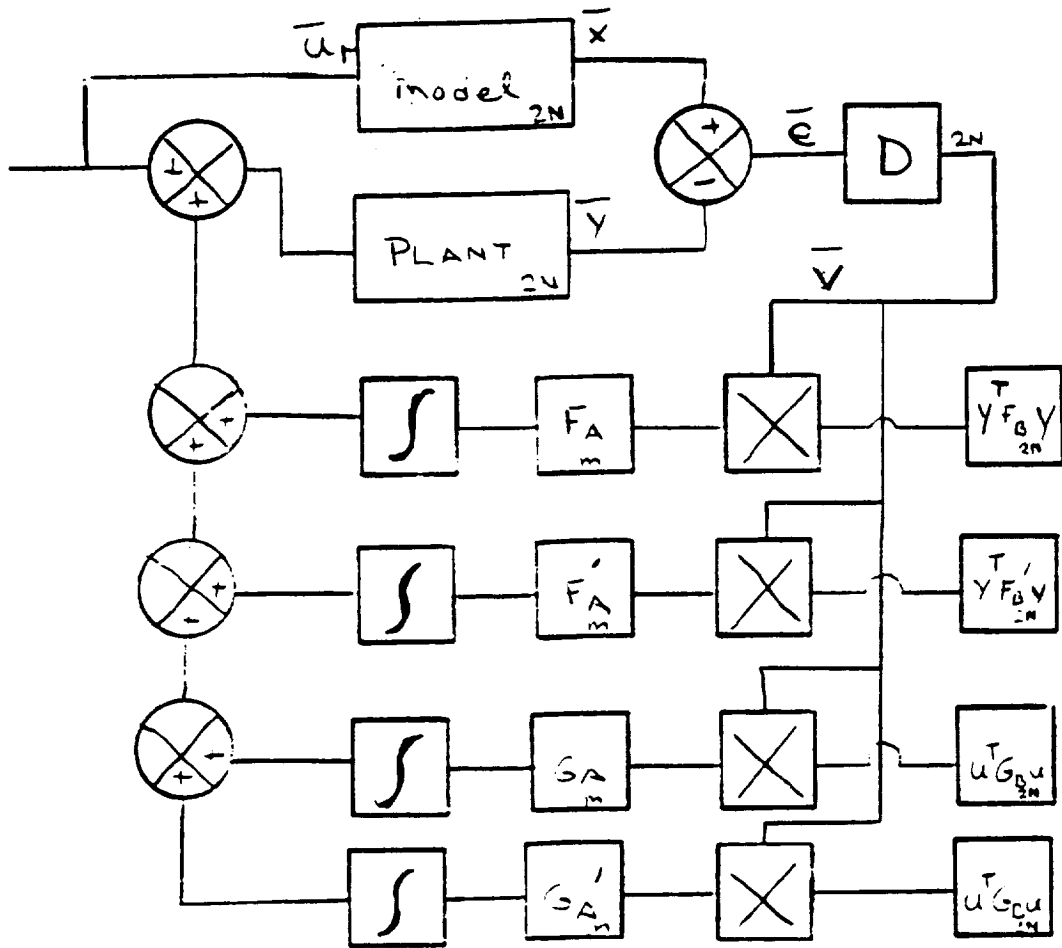
**Procedure:**

Find largest possible family of adaptation laws assuring stability, select specific adaptation law for particular application.

**Methods:**

**Hyperstability and Positivity Concepts**

## Control Approach



PI Adaptive Model Following Feedback

## Control Objectives

- Control designed for first five modes
- 2-10% damping required

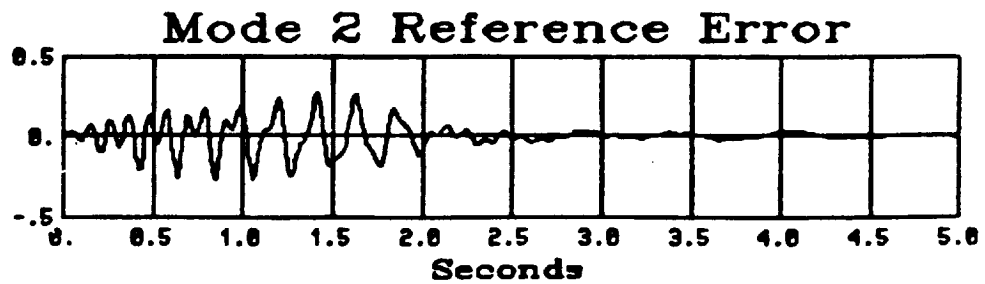
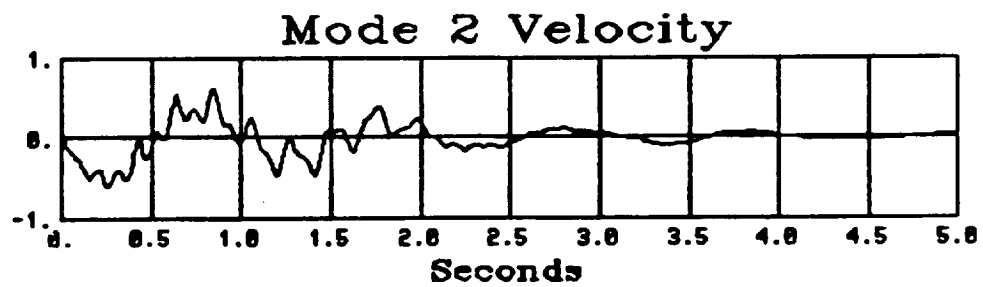
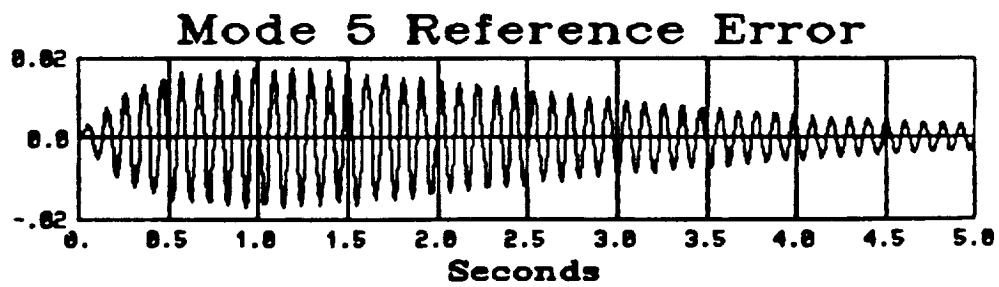
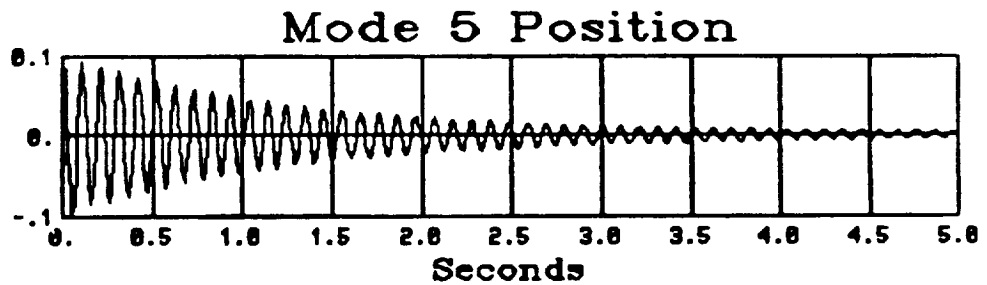
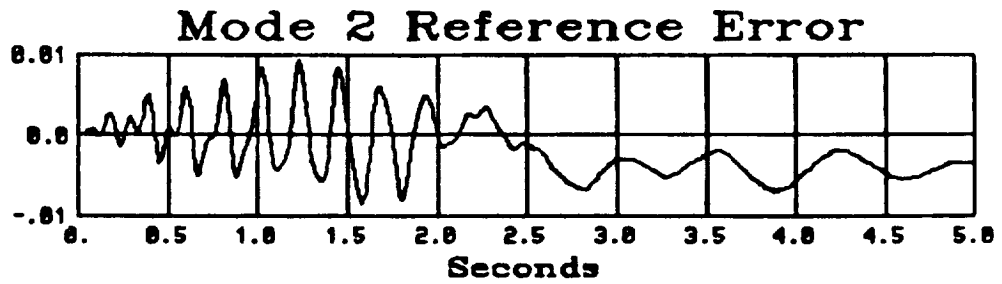
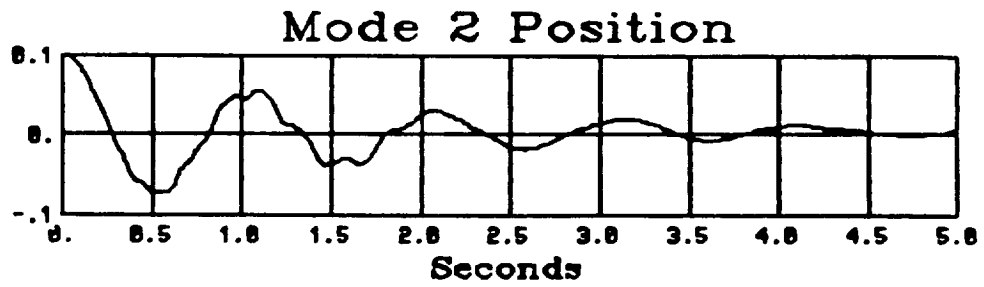
mode number	Frequency Hertz	Desired Damping
1	.964	10%
2	.964	10%
3	7.17	2%
4	7.51	2%
5	9.6	2%

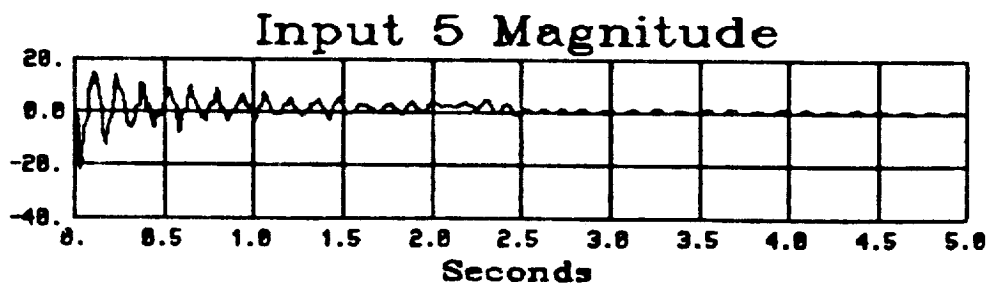
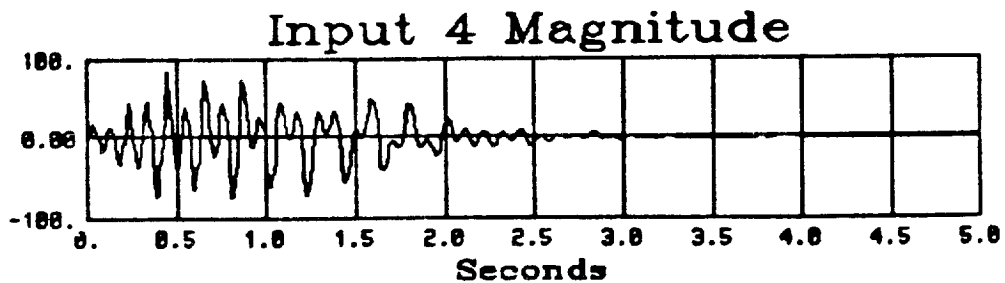
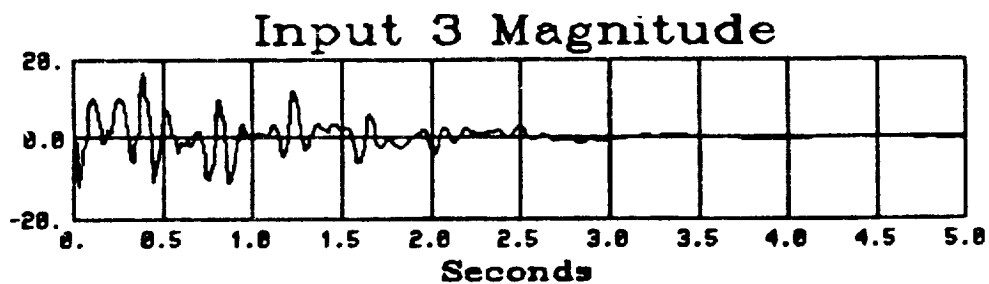
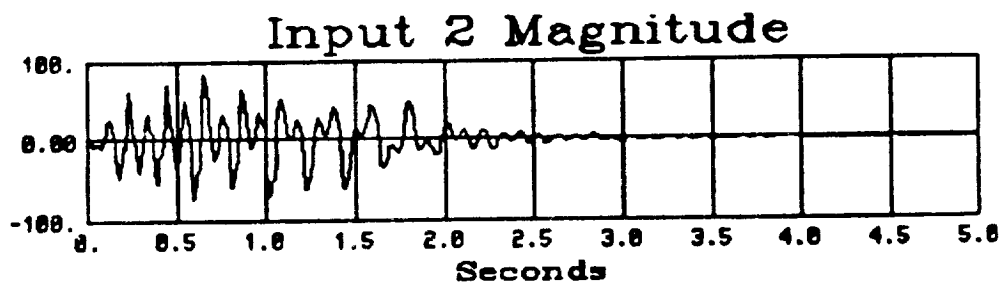
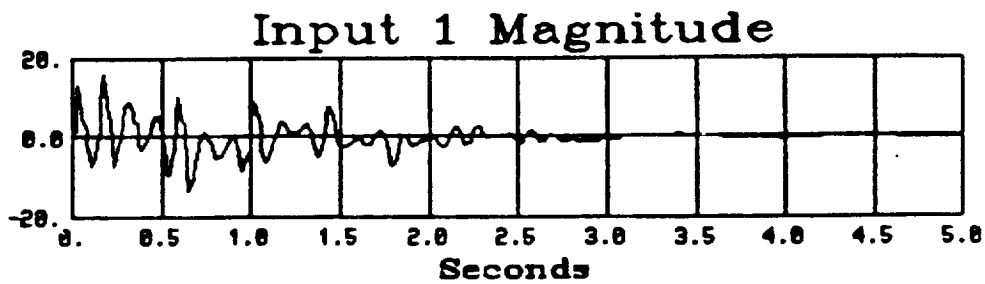
- Effects of actuator dynamics not included

mode number	Frequency Hertz	Mode Type
1	.964	x - y
2	.964	y - z
3	7.17	plate
4	7.51	torsion
5	9.6	y - z
6	9.8	x - z
7	10.2	y - z
8	12.1	mix
9	16.08	mix
10	16.8	mix

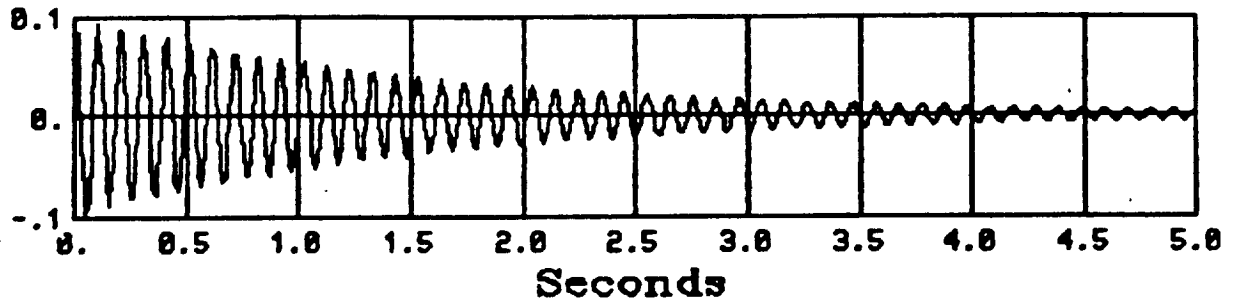
117



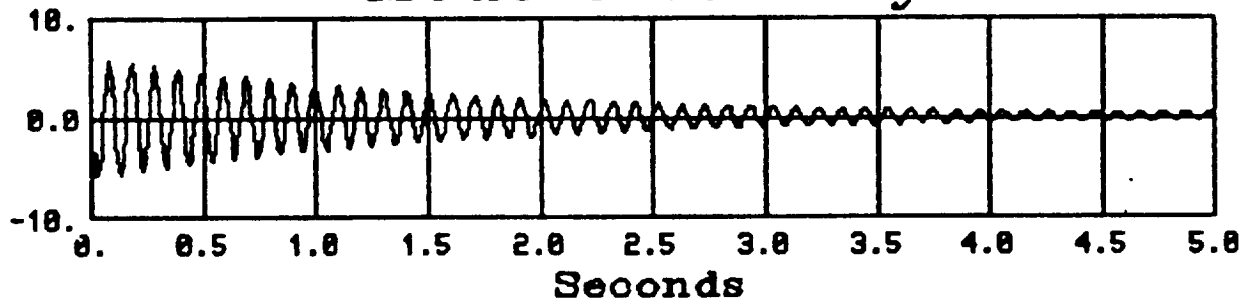




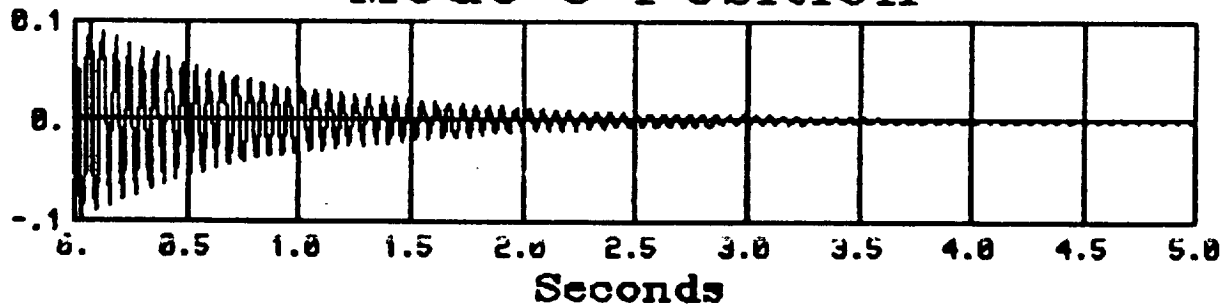
Mode 6 Position



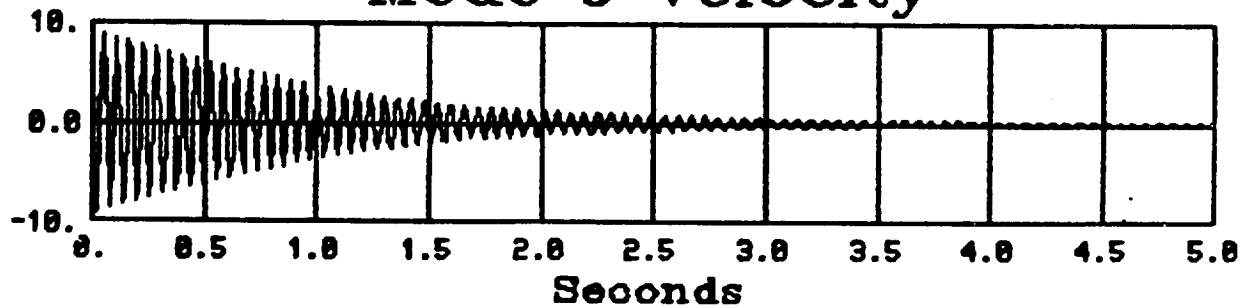
Mode 6 Velocity



Mode 9 Position



Mode 9 Velocity



## **Outlook**

- **Effects of Actuator Dynamics**
- **Refinement of STAC**
- **System Identification**
- **Experimentation**